MAY 1973

HEWLETT-PACKARD JOURNAL

A Pocket-Sized Answer Machine for Business and Finance

This new nine-ounce, battery-powered calculator replaces most commonly used financial tables, such as compound interest, annuities, and bonds. It's also a 200-year calendar.

by William L. Crowley and Francé Rodé

THE WORLD OF BUSINESS AND FINANCE DEPENDS ON THE FLOW OF MONEY — how much is flowing now and over a period of time, the value that people place on it, the expectations and (requirements) of it in the future, and the evaluation of it over some past history.

Accurate measurement of money as related to time is of vital importance to the business and financial community. However, accurate measurements have been difficult because extensive computational and mathematical abilities are required for many critical problems. But now, financial and business people can solve these difficult problems easily with a new tool, the HP-80 Pocket Calculator (Fig 1).

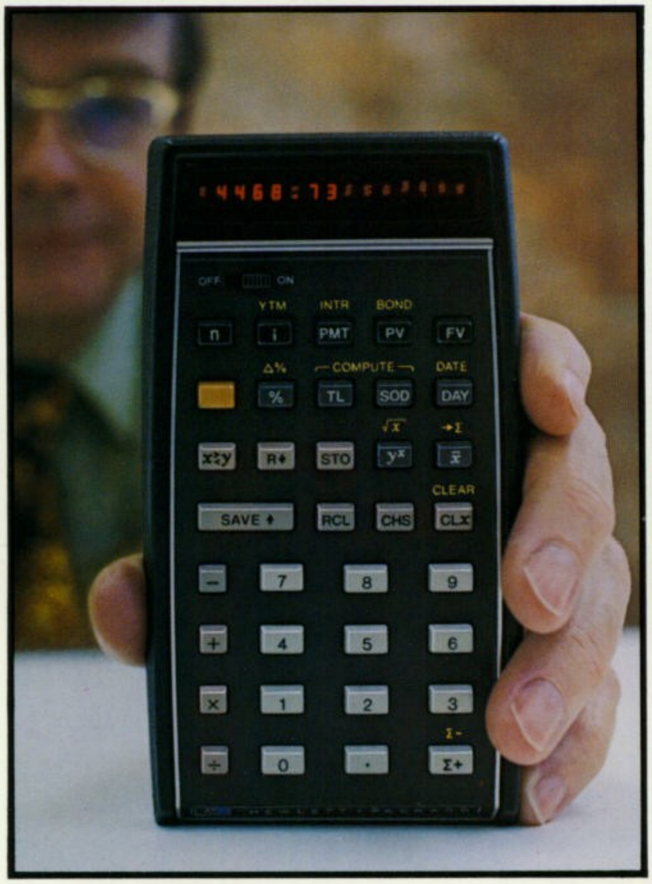


Fig. 1. *The HP-80 has a 10-digit display, plus sign and two-digit exponent. It weighs nine ounces and operates from rechargeable batteries or the ac line.*

The HP-80 is a preprogrammed computer/calculator designed specifically for financial and business calculations. The HP-80 looks very much like the HP-35, HP's first pocket calculator, which is designed for engineering and scientific work.[1] In fact, the HP-80 and the HP-35 operate the same arithmetically. But the design objectives for the HP-80 were less technical and more problem oriented. We wanted to produce an "answer machine," not an "equation-solver." This meant that the power of the basic HP-35 hardware had to be harnessed in such a way that the real complexity of solving many problems would be invisible to the user. In the final design, the user need only enter the parameters of the problem into the HP-80 and press a key for his answer.

Within the restrictions of the HP-35's architecture and with the help of colleagues on Wall Street and in the various money markets (as well as a little *sensum intestino*), over 30 hard-wired programs were implemented in the HP-80. These programs essentially replace all of the commonly used financial tables, such as compound interest, annuities, bonds, and so on. What's more, other difficult problems, such as time-series linear regression analysis and standard deviation, are made extremely simple. Answers are displayed to as many as six decimal places — more than are given by many tables. This increased precision is often valuable in bond yield and other critical problems.

One of the more intriguing features of the ma-

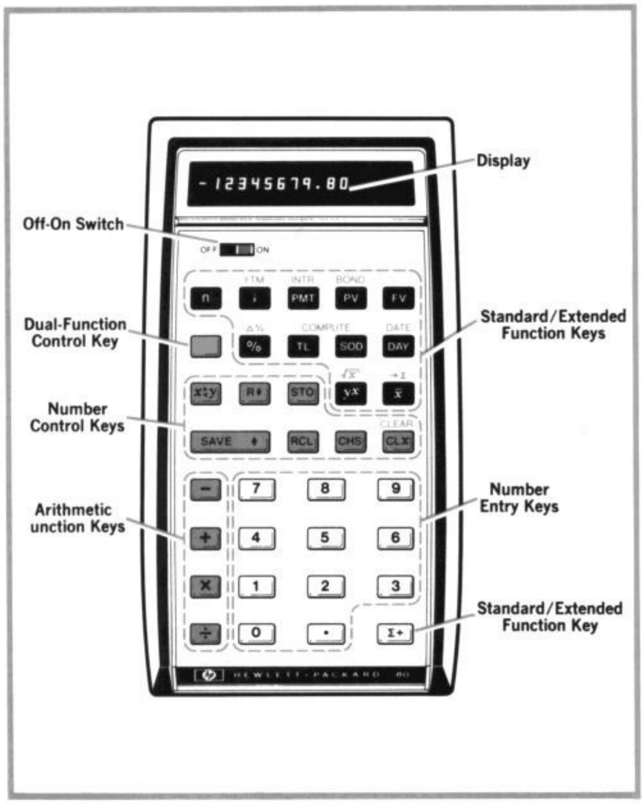


Fig. 2. *HP-80 keyboard layout. The gold key converts eleven other keys to their alternate (extended) functions, shown in gold above the keys.*

Cover: *"Ray Hutchinson at the Pacific Stock Exchange" is a familiar sign-off to listeners of San Francisco radio station KCBS. Art Director Arvid Danielson got the idea for this cover photo of the HP-80 Business Pocket Calculator while driving to work one day listening to the radio, and KCBS business editor Hutchinson and the Pacific Stock Exchange kindly agreed to pose for us.*

In this issue:

chine is its built-in 200 year calendar. Principally used in practical applications such as note and bond problems (for example, finding a bond yield between any two dates), it is also capable of such "extra-curricular" problems as finding the day of the week on which you were born or the number of days you've been on planet Earth.

The Keys and Their Uses

Fig. 2 shows the HP-80 keyboard layout and nomenclature. The functions of the keys are as follows.

n	The number of time periods pertaining to a particular problem (i.e., the number of days, months, years, etc.).
YTM / i	The interest rate as it pertains to a particular problem, expressed as a percent. Use of this key in the extended mode (using the GOLD key prior to this key) refers to yield-to-maturity calculations for bonds.
INTR / PMT	The payment or installment portion of an annuity or the coupon rate of a bond. Extended use of this key refers to the interest payment of a note.
BOND / PV	The present value or principal as it pertains to a particular problem. Extended use of this key refers to the present value or price of a bond.
FV	The future value as it applies to a given problem.
(GOLD key)	GOLD colored key converts other keys to their extended or alternate function.
Δ% / %	The percentage amount of some base number (e.g., calculates 5% of $50.00). Extended use of this key will calculate the percent difference between two values.
TL	Allows simple calculation of trend lines (by the least-squares linear regression method).
SOD	Computes amortization schedules using the sum-of-the-digits method. Can use either months or years for finance charge or depreciation.
DATE / DAY	Calculates the difference in calendar days between two dates. Extended use of this key finds a specific date given the number of days.
x⇄y	Exchanges the contents of the X-location with that of the Y-location.

R↓ Rotates the "stack" of stored values *down* one location each time this key is pressed.*

STO Stores a given value into the constant storage location (a separate storage register not in the stack).

√x / yˣ Raises a positive number to any power. Extended use of this key finds the square root of any positive number.

→Σ / x̄ Calculates the mean (arithmetic average) of a series of values previously entered by the summation key Σ+. Extended use of this key allows the user to return to the summing mode for additional observations.

SAVE↑ Stores a value into the calculator (*not* in constant storage) so that it can be operated on by another value and/or function.

RCL Recalls the value stored in the constant storage location.

CHS Changes the sign of the number shown in the display.

CLEAR / CLx Clears the display of whatever is shown. Extended use of this key clears the entire stack (but not the constant storage location).

Σ- / Σ+ Sums a series of numbers with the intention of determining a mean and standard deviation. Extended use of this key allows any value to be taken away from the sum, as in the case of an erroneous entry.

Readers familiar with the HP-35 will notice that the SAVE↑ key performs the same function as the HP-35's ENTER↑ key, and that the y^x key does the same thing as the x^y key on the HP-35, but in reverse order. Also, the HP-35 doesn't have the unlabeled gold key. The gold key's purpose is to serve as a shift or control key to execute a function labeled in corresponding gold print above eleven of the HP-80 keys.

Entering an Interest Problem

Where possible the entry of problem parameters has been designed to be as simple and as natural as possible. For instance, the top row of five keys (replacing the basic financial tables commonly used in business and commerce) allows the user to enter his data in a natural, left-to-right sequence.

As an example, consider the following problem. What is the effective compounded rate of growth required to double the worth of an investment over 8¼ years? To solve for "i," the interest rate per period, simply enter the known parameters as follows (assume the present value of the investment is $1 and the future value is $2):

8.25 [n] 1 [PV] 2 [FV] [i] → 8.76 (%)

Any of the five elements of a compound interest problem can be found by simply entering the known values in left-to-right fashion and pressing the key bearing the symbol corresponding to the unknown value. In this way, we have tried to reduce the complexity of a financial concept to the simplicity of a single function.

*Like the HP-35, the HP-80 has a four-register operational stack. The four stack registers are named X, Y, Z, and T. The display always shows the number in the X register. Pressing the SAVE↑ key moves the number in X into Y, the number in Y into Z, and the number in Z into T. The number originally in T is lost. The R↓ key moves the number in T into Z, the number in Z into Y, the number in Y into X, and the number in X into T.

SPECIFICATIONS
HP-80 Pocket Calculator

BASIC HP-80 CAPABILITIES

1. Add
2. Subtract
3. Multiply
4. Divide
5. Constant storage
6. Selective round-off
7. Percent calculation
8. Percentage difference
9. Square root
10. Exponentiation
11. Running total (summation)
12. Mean (arithmetic average)
13. Standard deviation
14. Number of days between two dates } Automatically checks for
15. Future date given number of days } improper date entries
16. Future value of an amount compounded
17. Present value of an amount compounded
18. Effective rate of return for compounded amounts
19. Number of periods required, given rate, principal and future value
20. Future value of an annuity
21. Present value of an annuity
22. Effective rate per period, given the future value and payment of an annuity
23. Effective rate per period, given the present value and payment of an annuity
24. Payment or installment of an annuity given future value
25. Payment or installment of an annuity given present value
26. Number of periods given the rate, payment and future value of an annuity
27. Number of periods given the rate, payment and present value of an annuity
28. Add-on to effective annual rate conversion
29. True effective annual yield
30. Linear regression (trend-line) forecasting
31. Sum-of-the-years digits depreciation amortization
32. Sum-of-the-months digits finance charge amortization
33. Discounted cash flow analysis
34. Accumulated mortgage interest paid
35. Remaining principal on a mortgage
36. Accrued interest (360 and 365 day year)
37. Discounted notes (360 and 365 day year)
38. Effective yield on discounted notes (360 and 365 day year)
39. Bond price
40. Yield-to-maturity of a bond

PRICE IN U.S.A.: $395.00

MANUFACTURING DIVISION: ADVANCED PRODUCTS DIVISION
10900 Wolfe Road
Cupertino, California 95014

Examples of HP-80 Solutions

Percent Difference Between Two Numbers

To find the percent difference between two numbers, enter the base number and press [SAVE ↑]. Enter the second number, press [ ] [Δ% / %]

For example,

ENTER:	SEE DISPLAYED:
60 [SAVE ↑] 240 [ ] [%] ⟶	300.00 % (240 is 300% greater than 60)

Day of the Week

Suppose today is Monday, January 22, 1973, and you wanted to find out what day it was when the stock market crashed on October 29, 1929.

ENTER:	SEE DISPLAYED:
10.291929 [SAVE ↑] 1.221973	
[DAY] [SAVE ↑] 7 [÷] ⟶	2255.86
2255 [−] 7 [×] ⟶	6.00 day factor

Today is Monday; count six days back to get Black Tuesday.

Accumulated Interest Between Two Points, Remaining Principal

You have a 30-year, 8% mortgage on your house in the amount of $40,885 which you obtained 4⅓ years ago. Monthly payments are $300. What is the amount of interest paid during the taxable year just ended starting at payment 40 and ending at payment 52? What is the unpaid principal?

Solution

ENTER:	SEE DISPLAYED:
40 [STO] 52 [n] 360 [n]	
8 [SAVE ↑] 12 [÷] [i] 300 [PMT] [Σ+] →	$ 3154.48 interest
[x⇄y] ⟶	$ 39186.73 remaining principal

Discounted Cash Flow Analysis

You are offered an investment opportunity for $100,000 at a capital cost of 10% after taxes. Will this investment be profitable based on the following cash flows?

Year	Cash Flow
1	$34,000
2	$27,500
3	$59,700
4	$ 7,800

Solution

ENTER:	SEE DISPLAYED:
10 [i] 100000 [CHS] [PV] 34000 [PV] [Σ+] →	$ −69090.91 current net present value
27500 [PV] [Σ+] ⟶	$ −46363.64 current net present value
59700 [PV] [Σ+] ⟶	$ −1510.14 current net present value
7800 [PV] [Σ+] ⟶	$ 3817.36 final value is positive deal is OK

A Bond Problem

The general equation for the price of a corporate bond in the United States is:

$$p = -100\left(\frac{a}{2}\right)\left(\frac{180-b}{180}\right) + \left[\sum_{k=1}^{c} \frac{100\left(\frac{a}{2}\right)}{\left(1+\frac{m}{2}\right)^{\left\{k-1+(b/180)\right\}}}\right] + \frac{100}{\left(1+\frac{m}{2}\right)^{\left\{c-1+(b/180)\right\}}}$$

where:

p = price of bond expressed as a percent
a = annual interest rate payable semi-annually; the interest is expressed as a decimal
b = number of days before the next coupon is payable
c = number of coupons payable between the present and maturity
m = annual yield to maturity, compounded semi-annually; the yield is expressed as a decimal.

One can imagine how difficult it would be to solve for the yield to maturity, m! With the HP-80, the solution is simple. For example, what is the yield to maturity of a bond bearing a 5% coupon, maturing in 5 years and selling at 94½?

To obtain yield to maturity, enter the following in left-to-right sequence and see the answer 6.30 (%) displayed.

5 [SAVE ↑] 365 [×] [n] 5 [PMT] 94.5 [PV] [ ] [i / YTM] ⟶ 6.30 (%)

Trend Lines

The HP-80 can be used to find the least-squares linear approximation to a time series and extrapolate it into the past or future. The technique is known as linear regression or trend-line analysis.

For example, using linear regression, calculate a projected value for sales in the seventh month if actual sales for the first six months are 486, 437, 506, 470, 532, 512.

HP-80 solution:

486 [TL] 437 [TL] 506 [TL] 470 [TL] 523 [TL] 512 [TL] [ ] [TL]

7 [n] [TL] ⟶ 524.20

Conventional solution:

$$Y_X = A + BX$$

where:

Y_X = value on the trend line at the point in time X
A = value of the trend line at the origin (X = 0)
B = amount of change in the trend line value per unit of time X
X = units of time

$$B = \frac{2 \sum_{K=1}^{n} KY_K - (n+1) \sum_{K=1}^{n} Y_K}{\frac{n(n^2-1)}{6}} = 10.06$$

$$A = \frac{1}{n} \sum_{K=1}^{n} Y_K - \left(\frac{n+1}{2}\right)(B) = 453.79$$

Then $Y_7 = 453.79 + (10.06)(7) = 524.20$

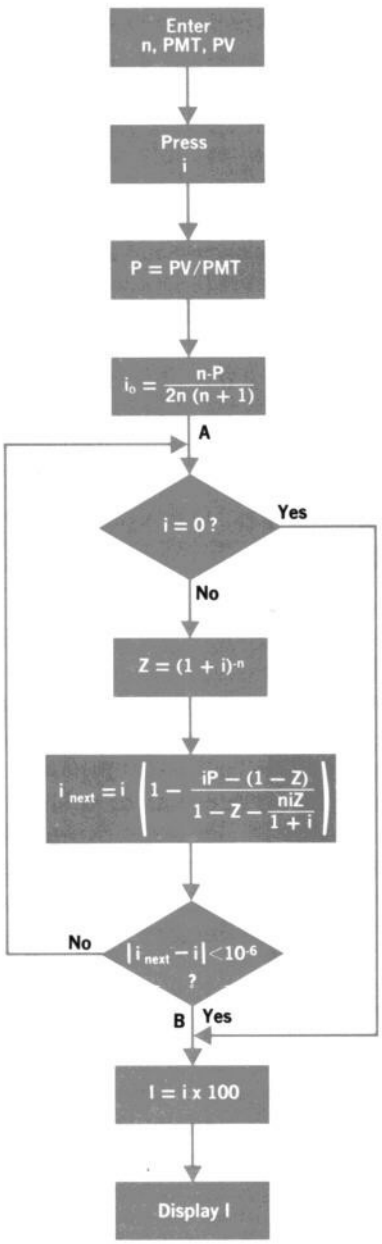
Other Capabilities

A list of the HP-80's basic capabilities appears on page 4. More complex problems can be solved by combining these basic capabilities. Additional examples of problem solutions are on page 5.

Inside the HP-80

Architecturally, the HP-80 is essentially the same as the HP-35[1], so instead of describing its architecture completely we'll highlight only the differences.

In many respects the HP-80 is more like a computer than a calculator. It has a microprogrammed instruction set that is essentially the same as the



Step Nos.	ROM Address	ROM Code	ROM Subroutine Addresses	Labels	Program Statements
122	L03172:	.1.....1.1	-> L3101		JSB DR3
123	L03173:	111.1.111.			A EXCHANGE C[W]
124	L03174:	.11.1.1..1	-> L3152		JSB SUB
125	L03175:	..1.1.1...			C EXCHANGE M
126	L03176:	...1.1.1.1	-> L3025		JSB DR2
127	L03177:	.1...11..1	-> L3106		JSB ONE
128	L03200:	.11.1.11.1	-> L3153		JSB ADD
129	L03201:	11..1.111.			A EXCHANGE B[W]
130	L03202:	111..1...1	-> L3344		JSB MPY
131	L03203:	.11.1.11.1	-> L3153		JSB ADD
132	L03204:	1.1.1.1...			M -> C
133	L03205:	111.1.111.			A EXCHANGE C[W]
134	L03206:	111...11.1	-> L3343		JSB DIV
135	L03207:	..1.1.1...			C EXCHANGE M
136	L03210:	1..11..11.		FVR44 :	IF A[M] >= 1
					THEN GO TO FVR50
137	L03211:	1...1.1111	-> L3213		GO TO FVR46
138	L03212:	.1.111..11	-> L3134	FVR50 :	DOWN ROTATE
139	L03213:	11..1.1...			C EXCHANGE M
140	L03214:	..1.1.1...			STACK -> A
141	L03215:	.11.1.1...			JSB ONE
142	L03216:	.1...11..1	-> L3106		JSB ADD
143	L03217:	.11.1.11.1	-> L3153		C -> STACK
144	L03220:	.1..1.1...			C -> STACK
145	L03221:	.1..1.1...			B -> C[W]
146	L03222:	..1...111.			C EXCHANGE M
147	L03223:	..1.1.1...			JSB XTY
148	L03224:	.....1...1	-> L3004		DOWN ROTATE
149	L03225:	11..1.1...			C -> STACK
150	L03226:	.1..1.1...			JSB MPY
151	L03227:	111..1...1	-> L3344		B -> C[W]
152	L03230:	..1...111.			JSB DR2
153	L03231:	...1.1.1.1	-> L3025		JSB DIV
154	L03232:	111...11.1	-> L3343		JSB DR2
155	L03233:	...1.1.1.1	-> L3025		JSB ONE
156	L03234:	.1...11..1	-> L3106		JSB SUB
157	L03235:	.11.1.1..1	-> L3152		M -> C
158	L03236:	1.1.1.1...			JSB DIV
159	L03237:	111...11.1	-> L3343		JSB DR2
160	L03240:	...1.1.1.1	-> L3025		JSB SUB
161	L03241:	.11.1.1..1	-> L3152		STACK -> A
162	L03242:	.11.1.1...			C -> STACK
163	L03243:	.1..1.1...			B EXCHANGE C[W]
164	L03244:	1...1.111.			JSB ADD
165	L03245:	.11.1.11.1	-> L3153		B EXCHANGE C[W]
166	L03246:	1...1.111.			DOWN ROTATE
167	L03247:	11..1.1...			JSB DIV
168	L03250:	111...11.1	-> L3343		M -> C
169	L03251:	1.1.1.1...			JSB MPY
170	L03252:	111..1...1	-> L3344		B EXCHANGE C[W]
171	L03253:	1...1.111.			C EXCHANGE M
172	L03254:	..1.1.1...			JSB ADD
173	L03255:	.11.1.11.1	-> L3153		C EXCHANGE M
174	L03256:	..1.1.1...			STACK -> A
175	L03257:	.11.1.1...			C -> A[W]
176	L03260:	.11...111.			JSB TEN6
177	L03261:	1111..11.1	-> L3363		IF A[XS] >= 1
					THEN GO TO FVR46
178	L03262:	1..1111.1.			GO TO FVR44
179	L03263:	.1.111..11	-> L3134	PMT42	IF S11 # 1
					THEN GO TO PV46
180	L03264:	1...1...11	-> L3210	FV46	IF S10 # 1
					THEN GO TO PV49
181	L03265:	1.11.1.1..			
182	L03266:	.1...11111	-> L3107		
183	L03267:	1.1..1.1..			
184	L03270:	.1..1..111	-> L3111		

Fig. 3. *HP-80 algorithm for finding interest rate given present value, payment, and number of periods. Also shown is a partial listing of the program that implements the algorithm.*

HP-35's except that trigonometric functions are excluded. But the HP-80 also has another level of programming that uses the basic functions (+, −, ×, ÷, y^x, ln x, etc.) as subroutines. Also, some of the more frequently used register manipulation routines are incorporated as subroutines in the HP-80 to save memory space.

An example of HP-80 programming is shown in Fig. 3. The flow chart shows the algorithm used to solve for the interest i in the equation

$$PV = PMT\,\frac{1-(1+i)^{-n}}{i}$$

See Appendix A for the derivation of the algorithm.

The equation is solved by an iterative technique. The program listing shown in Fig. 3 is the portion of the program that implements the iteration sequence (that is, the part between A and B in the flow chart).

Both the microcode and the higher-level code in the HP-80 are implemented in read-only memories (ROMs). Where the HP-35 had three ROMs the HP-80 has seven. The seven ROMs are mounted on a hybrid circuit to form a single assembly (Fig. 4). The hybrid approach makes final assembly easier and allows use of a less expensive circuit mounting board.

Accuracy

Accuracy of the HP-80 varies according to the operation being performed.

Arithmetic operations (+, −, ×, ÷) are accurate to 10 digits ± 1 count; that is, the value of the 10th digit could be 1 less or 1 more than the value displayed.

Special operations solved through an iterative method *(bond yields, effective yield of an annuity, and add-on to effective annual rate conversion problems)* use numerical analysis techniques formerly requiring computers or large-scale programmable calculators. These operations are accurate to 5 digits.

All other operations are accurate to 9 digits ±1 count; the value of the 9th digit could be 1 less or 1 more than the value displayed.

Bond yield calculations may result in slightly reduced accuracy if the problem is an unusual case such as calculating a 50% yield-to-maturity for a 2% bond. For example, the yield-to-maturity on a 2% bond, maturing in 3 years, 3 months and selling at 26.481133 is 50.00%. The answer displayed is 49.99%. The formula for determining error magnitude is: percent calculation error for bond yield $\leq 2 \times$ yield (actual) $\times$ coupon $\times 10^{-4}$. The operating limits for the bond yield algorithm are defined such that the price of the bond must be above 20 and below 5,000, and coupon must be greater than 1/8% and less than the price of the bond (expressed as a percent of par).

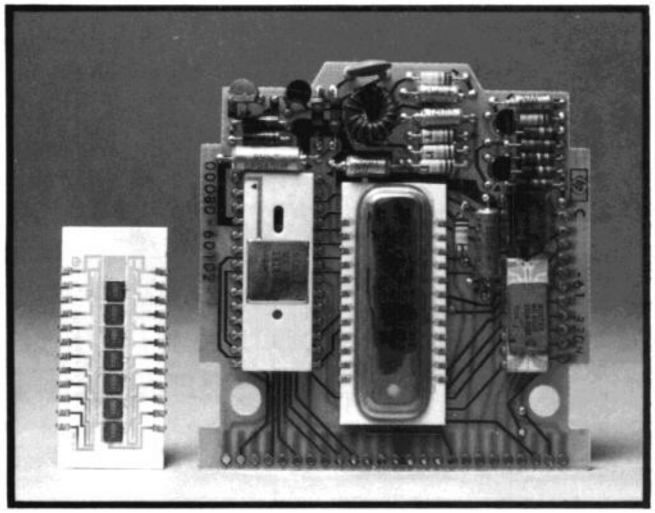


Fig. 4. *HP-80 programs are stored in seven read-only-memory chips mounted on a hybrid circuit.*

Error Indicators

Any computation or data entry resulting in a magnitude greater than or equal to 10^{100}, or less than 10^{-99}, triggers an error signal indicated by a blinking display.

If a calculation is attempted that contains a logical error—say division by zero—an error signal is triggered and a blinking display appears. To reset, press CLX. Examples of logical errors are division by zero, y^x where $y \leq 0$, and date/day calculations where the range is exceeded, a date entered doesn't exist, or date-entry conventions aren't followed.

Acknowledgments

The individuals who helped with the HP-80 development did so with a spirit of immense teamwork. Particular appreciation is given to the following: Barney Oliver for his insistence and guidance on fundamental product excellence that only a mind of his scope and experience can provide. Paul Stoft and Tom Whitney for their assistance in providing technical aid and coordination of resources. Sandy Walker for providing valuable assistance on many of the "tricky" algorithms implemented in the HP-80. Dave Cochran for his assistance in implementing and consulting on some very clever microprogramming. Darrel Lauer for his superb job in implementing a keyboard set that defied many physical limitations. Ken Peterson and the Loveland Division IC department for the hybrid ROM circuit. The HP finance and accounting personnel who provided helpful comments and suggestions. And Bill Hewlett, who gave us the trust and freedom that allowed all of this to happen.

APPENDIX A

A Typical HP-80 Algorithm

Some of the functions incorporated into the HP-80 are implicit functions of i and must be solved by iterative techniques. Since we are register and ROM limited, the algorithms had to be as compact and efficient as possible.

For a variety of reasons the best technique to try is the Newton-Raphson one[2] and this was used for the annuity functions. Take the equation

$$PV = PMT \frac{1 - (1 + i)^{-n}}{i}$$

We wish to solve for i knowing the rest of the variables. Rewriting this as a function of i,

$$f(i) = \frac{PV}{PMT} - \frac{1 - (1 + i)^{-n}}{i} \qquad (1)$$

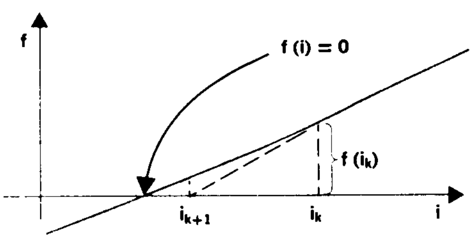
and we are solving for $f(i) = 0$. First we sum the first three terms of the binomial expansion for $f(i)$ and solve for i. This gives us i_o, the starting value. Thus

$$i_o = \frac{2(n - PV/PMT)}{n(n + 1)}.$$

We now look for some relationship that will give us $i_1, i_2, \ldots i_k, i_{k+1}$ successively such that for some k

$$| i_k - i_{k+1} | < \text{error limit we require}$$

The diagram shows that if $f(i_{k+1})$ is very small, the following relationship holds for $df(i_k)/di_k$.



$$f'(i_k) \approx \frac{f(i_k)}{i_k - i_{k+1}}$$

so that

$$i_{k+1} = i_k - \frac{f(i_k)}{f'(i_k)} \qquad (2)$$

Basically we slide down the curve sawtoothing into the solution with this relationship. If $f(i)$ is very steep we may take a lot of sawtoothing into the required accuracy. Converting this to an algorithm is just:—

$i \leftarrow i_o$	feed into i the starting value
1: $i_{next} \leftarrow i - f(i)/f'(i)$	compute next i value
$\| i_{next} - i \| < 10^{-6}$ (say)	check convergence to accuracy required
If yes then exit	
If no then $i \leftarrow i_{next}$ and go to 1 to compute $(i_{next})_{next}$.	

This is the Newton-Raphson technique.

For the function $f(i)$ defined in equation 1, i_{next} is derived as follows:

$$f'(i) = -\frac{n(1 + i)^{-n}}{i(1 + i)} + \frac{1 - (1 + i)^{-n}}{i^2}$$

Let $P = PV/PMT$ and $Z = (1 + i)^{-n}$.
Then

$$i_{next} = i - \frac{f(i)}{f'(i)} = i - \frac{\dfrac{iP - (1 - Z)}{i}}{\dfrac{1}{i^2}\left[(1 - Z) - \dfrac{niZ}{1 + i}\right]}$$

$$= i\left[1 - \frac{iP - (1 - Z)}{1 - Z - \dfrac{niZ}{1 + i}}\right] \qquad (3)$$

Fig. 3, page 6 shows the flow diagram and HP-80 program listing to solve equation 1 using the Newton-Raphson technique and i_{next} as defined by equation 3.

APPENDIX B

Principal HP-80 Equations

1. $FV = PV(1 + i)^n$
   FV = future value, PV = present value, n = number of periods, i = interest per period.
2. $FV = PMT \frac{(1 + i)^n - 1}{i}$
   PMT = payment
3. $PV = PMT \frac{1 - (1 + i)^{-n}}{i}$
4. $I_{J-K} = PMT\left[K - J - \frac{(1 + i)^{K-n}}{i} + \frac{(1 + i)^{J-n}}{i}\right]$
   I_{J-K} = interest from payment J to payment K
5. $PV_K = \frac{PMT}{i}\left[1 - (1 + i)^{K-n}\right]$
   PV_K = remaining balance after payment K
6. Discounted Note Calculations
   $D = \frac{FVni}{N} \qquad Y = \frac{ND}{n(FV - D)}$
   D = discount rate, Y = yield, N = 360 or 365 (usually 360 in U.S.A.)
7. Depreciation
   $dep_k = \frac{2(n - k + 1)}{n(n + 1)} PV$
   dep_K = depreciation for kth period using sum-of-digits (SOD) method.
8. Bond Price*
   For $n \geq 182.5$ (days between purchase and maturity)
   $p = 100(1 + m/2)^{-n/182.5} + 100(a/i) \cdot [(1 + m/2)^J - (1 + m/2)^{-n/182.5}] - 100(a/2)J$
   For $n \leq 182.5$
   $p = \frac{200 + 100a}{2 + nm/180} - (1 - n/180)100(a/2)$
   p = bond price (based on 360 or 365 day year, depending upon how n is computed), a = coupon rate expressed as a decimal on an annual basis, m = annual interest called *annual yield to maturity* expressed as a decimal, J = 1 − b/180, b = number of days before next coupon is payable.
   *Note: This is a more general form of the equation shown on page 5 .
9. Trend Lines
   $Y_X = A + BX$
   Y_X = value of trend line at point in time X, A = value at X = 0, B = slope of trend line.
   $B = \frac{2\sum_{K=1}^{n} KY_K - (n + 1)\sum_{K=1}^{n} Y_K}{n(n^2 - 1)/6}$
   $A = \frac{1}{n}\sum_{K=1}^{n} Y_K - \left(\frac{n + 1}{2}\right)B$
   n = number of time points for which data is available.
   K = 1, 2, . . . , n.
   Y_K = trend line value at Kth time point.
10. Standard Deviation
    $S_X = \sqrt{\frac{1}{n - 1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}$
    S_X = standard deviation of variable x, n = number of samples of x, $\bar{x}$ = mean value of x, i = 1, 2, . . . , n.
11. Discounted Cash Flow Analysis (Net Present Value)
    $NPV = \sum_{k=0}^{n} C_k(1 + i)^{-k}$
    NPV = net present value, i = discount rate expressed as a decimal, n = number of cash flows, C_k = cash flow in kth period, k = 1, 2, . . . , n.

References

1. T. M. Whitney, F. Rodé, and C. C. Tung, "The Powerful Pocketful: an Electronic Calculator Challenges the Slide Rule," Hewlett-Packard Journal, June 1972.
2. C. Jordan, "Calculus of Finite Differences," Chelsea Publishing Company, New York, 1965. Page 489.

Francé Rodé

Francé Rodé came to HP in 1962, designed counter circuits for two years, then headed the group that developed the arithmetic unit of the 5360 Computing Counter. He left HP in 1969 to join a small new company, and in 1970 he came back to HP Laboratories. He designed the arithmetic and register circuit and two of the special bipolar chips for the HP-35, HP's first pocket calculator, and he was project leader of the HP-80 development team, designing the hardware while Bill Crowley developed the algorithms. Francé holds the degree Deploma Engineer from Ljubljana University in Yugoslavia. In 1962 he received the MSEE degree from Northwestern University. When he isn't designing logic circuits he likes to ski, play chess, or paint.

William L. Crowley

Bill Crowley, HP's corporate cash manager, was half of the two-man team that developed the HP-80. Bill contributed the algorithms and Francé Rodé designed them into hardware. Bill received his BS degree in mechanical engineering in 1964 from Texas A & M University and his MBA degree in 1967 from the University of Texas. During the next 18 months he started two companies dealing in software system consulting and investment management, and developed and published a statistical model for security price forecasting. In 1969 he sold his businesses and joined HP, becoming cash manager 18 months later. Investments, computers, and financial history occupy a good deal of Bill's spare time, too, but occasionally he stops thinking about money and enjoys backpacking, ski touring, photography, and music.

Laboratory Notebook *

Thick Films Widen Attenuator Response

In designing an oscilloscope for high-frequency performance, the attenuator can be as much a problem as the amplifiers. The inevitable distributed capacitance and inductance work against obtaining both high impedance and accurate attenuation simultaneously over a broad range of frequencies. With conventional rotary switches and discrete resistors, 75 MHz has been about the limit for 1-megohm oscilloscope attenuators.

The attenuator in late-model HP high-frequency oscilloscopes overcomes this limit by using thick-film resistors and conductors deposited on an alumina substrate. The attenuator is switched by spring fingers that make contact to the conductors when depressed by plastic rods that are in turn actuated by cams on the switch shaft. Conductor paths are thereby shortened considerably and as a result, series inductance, the major cause of poor attenuator performance at high frequencies, is reduced as much as five times.

Accumulated stray capacitance, which usually seemed to amount to at least 24 pF at the input of conventional attenuators, is less than 13 pF. This doubles the frequency range over which high-impedance probing may be performed without serious loading of the circuits under test. Furthermore, circuit paths are much more uniform from attenuator to attenuator, reducing the need for excess compensation to take care of all variables.

The attenuator has 1-megohm input resistance. The physical design, however, also made it possible to include a switchable 50Ω termination at the input. Since the input, including the connector and switch, can be made part of a 50Ω line right up to the termination, this makes a clean 50Ω input impedance available for the oscilloscope. As a result, VSWR with the 50Ω termination switched in is only 1.2 at 100 MHz on all attenuator ranges, much better than that obtainable using an external termination. Accurate transition-time measurements on 50Ω circuits can thus be made with no risetime degradation attributable to the scope input. The 2-watt capability of the 50Ω termination allows input signals as large as 10 volts rms.

The entire attenuator is fabricated with only one conductor and two resistor screenings, one at 100k ohms per square and one at 30 ohms per square. The resistors are trimmed on the substrate to an accuracy of 0.5%, allowing the entire attenuator to be specified conservatively to an accuracy of ±2%. Temperature coefficient is 100 ppm/°C with tracking within 25 ppm/°C.

Instruments using this attenuator include the Model 1710A 150-MHz Portable Oscilloscope, the Models 1805A 100-MHz and 1808A 75-MHz Vertical Amplifier plug-ins for the 180-series Oscilloscope, and the Model 1834A 4-Channel 200-MHz Vertical Amplifier plug-in for the 183-series.

—Thomas Zamborelli
Colorado Springs Division

*Editors' note: "Laboratory Notebook" is a new editorial feature that describes individual developments of note. It will appear on an intermittent basis.